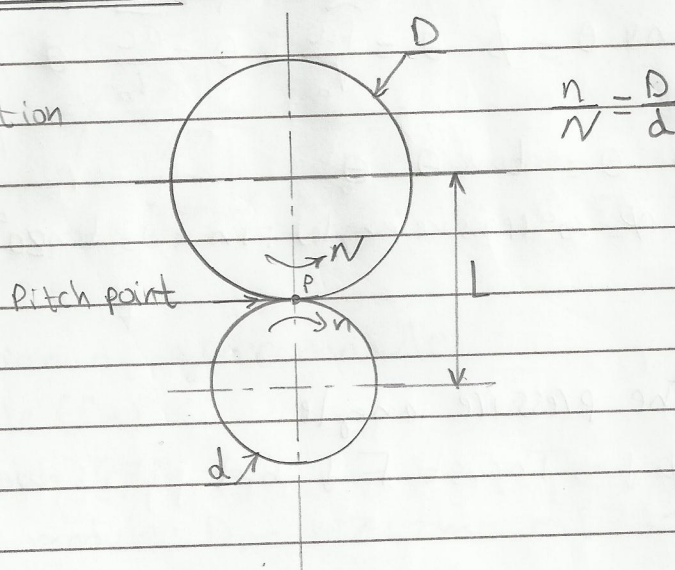


## 7. Gear measurement

### \* Friction wheels

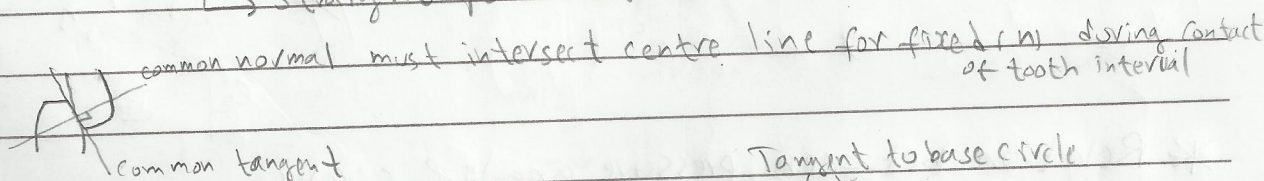
- For low power & high friction transmission



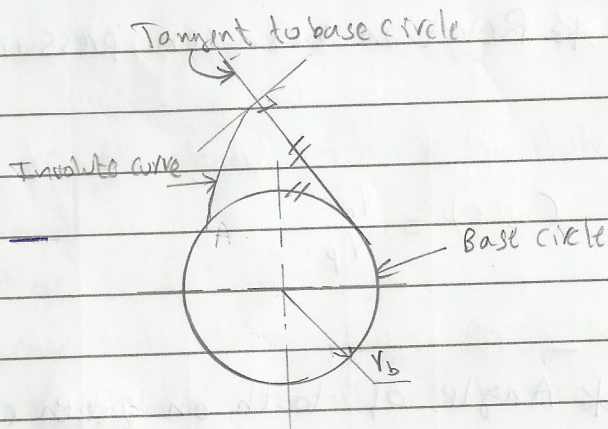
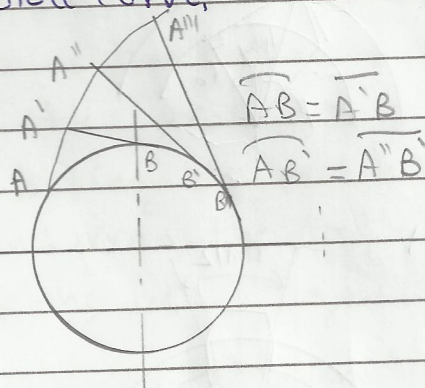
### \* Gears:

- Positive transmission in which high power is transmitted

- Tooth curves
  - Involute → Easier to manufacture and wear ☐
  - Cycloidal → Difficult to manufacture & strong (casting) ☐
  - Straight for rack



### \* The involute curve:



For any point on involute

- Circle a wire around a disk. Pull the wire and remove it as well so wire end draws involute
- Move a pencil around a disk to draw the involute curve
- Tangent on base circle ⊥ inv. curve
- Normal to involute curve is tangent to base circle



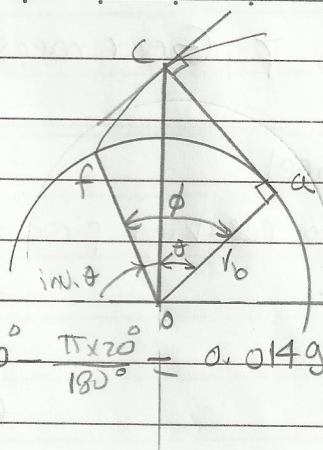
## \* Involute function:

From definition:  $\overline{ac} = \overline{af}$

$$\text{inv. } \theta = \phi - \theta = \frac{\widehat{fa}}{r_b} - \theta = \frac{\overline{ac}}{r_b} - \theta$$

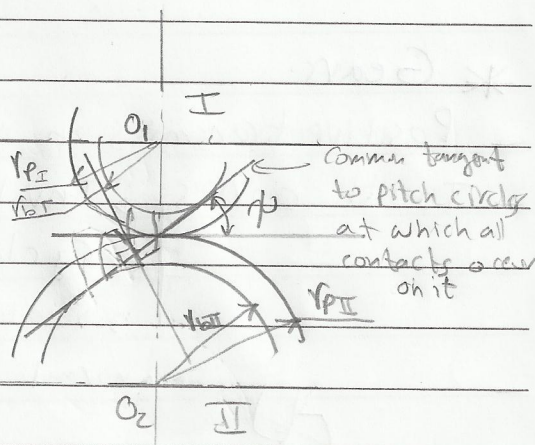
$$\text{inv. } \theta = \tan \theta - \theta$$

For  $\psi = 20^\circ$  (pressure angle):  $\text{inv. } \theta = \tan 20^\circ - \frac{\pi \times 20^\circ}{180^\circ} = 0.0149$



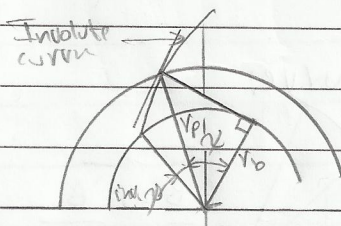
## \* The pressure angle

$\psi \downarrow \Rightarrow$  Thrust  $F \downarrow$  but it's weak



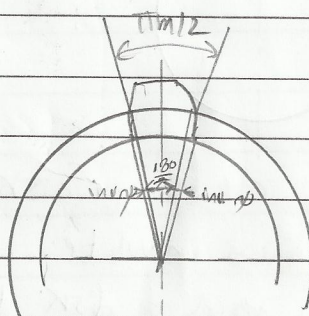
## \* Relation between pressure angle &amp; circles:

$$\cos \psi = \frac{r_b}{r_p}$$



## \* Angle of tooth on base circle:

$$\theta = \frac{180}{z} + 2 \text{ inv. } \psi$$



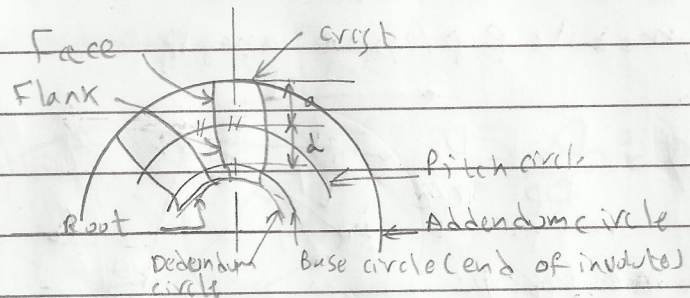


### \* Parameters of gear

$$a = m$$

$$d = 1.25m \text{ (for clearance)}$$

$$\text{Tooth depth} = a + d = 2.25m$$



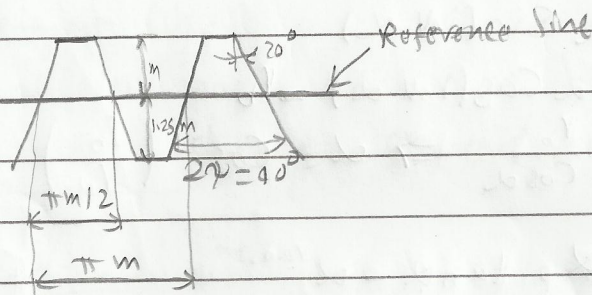
### \* Experimental prediction of gear module

1. Measure addendum circle ( $D_a$ )
2. Count the no. of teeth ( $Z$ )
3. Determine the gear module:  $D_a = mZ + 2m = m(Z + 2)$
4. Select the nearest standard module

### \* Involute rack:

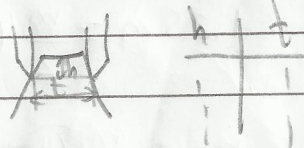
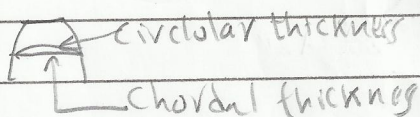
Tooth thickness (arc) at pitch circle

$$= \frac{\pi D_p}{2Z} = \frac{\pi m Z}{2Z} = \frac{\pi m}{2}$$



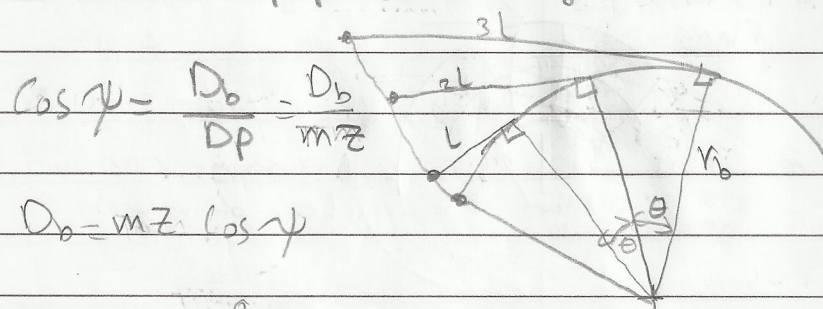
### \* Experiment: check the form of the tooth flank (involute)

1. Determine the module of gear
2. Draw the profile of flank of tested gear by
  - A. Projecting the profile of gear tooth flank using projector
  - B. Measuring the tooth thickness (Chordal) at different tooth depths using gear tooth vernier, T.M.M, etc.





3. Draw the theoretical involute curve for the given module of pressure angle



$$\cos \psi = \frac{D_b}{D_p} = \frac{D_b}{mz}$$

$$D_b = mz \cos \psi$$

$$L = r_b \times \frac{\theta \pi}{180^\circ}$$

$$\theta \rightarrow S^\circ$$

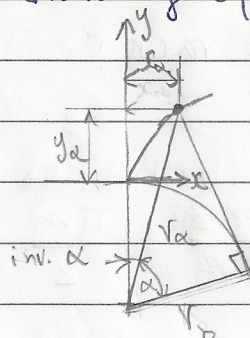
4. Compare the actual profile with the theoretical one

\* Drawing of involute curve without drawing of base circle;

$$x_\alpha = r_\alpha \sin(\text{inv. } \alpha)$$

$$y_\alpha = r_\alpha \cos(\text{inv. } \alpha) - r_b$$

$$r_\alpha = \frac{r_b}{\cos \alpha} \Rightarrow \alpha = \cos^{-1} \left( \frac{r_b}{r_\alpha} \right)$$



$$\text{inv. } \alpha = \tan \alpha - \alpha^{\text{rad.}}$$

$$\frac{\cos^{-1} r_b}{r_\alpha} \tan \alpha^{\text{rad.}}$$

| $r_i$ | $\alpha$ | $\text{inv. } \alpha$ | $x_\alpha$ | $y_\alpha$ |
|-------|----------|-----------------------|------------|------------|
| $r_d$ |          |                       |            |            |
| $r_p$ |          |                       |            |            |
| $r_a$ |          |                       |            |            |

$$r_d = \frac{mz}{2} - 1.25m = \frac{m}{2} (z - 2.5)$$

$$r_p = \frac{mz}{2}$$

$$r_a = \frac{mz}{2} + m = \frac{m}{2} (z + 2)$$



**\* Chordal thickness:**

(tc) Chordal thickness of pitch circle

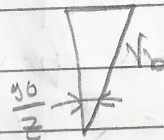
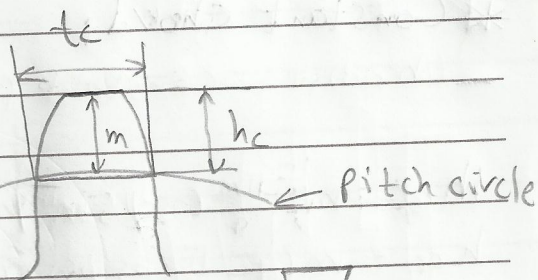
(hc) Chordal height

$$t_c = 2 r_p \sin \left( \frac{90}{z} \right) = 2 \left( \frac{mz}{2} \right) \sin \left( \frac{90}{z} \right)$$

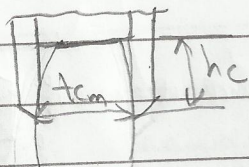
$$t_c = mz \sin \left( \frac{90}{z} \right)$$

$$h_c = m + \left[ r_p - r_p \cos \left( \frac{90}{z} \right) \right]$$

$$h_c = m + \frac{mz}{2} \left[ 1 - \cos \left( \frac{90}{z} \right) \right]$$

**Experimental procedure:**

1. Measure outer diameter
2. Count no. of teeth
3. Determine the module
4. Approximate to the nearest standard module
5. Calculate the chordal height ( $h_c$ )
6. Set the vertical vernier of gear tooth vernier on ( $h_c$ ) value
7. Set it on the tooth tip as shown in figure & measure its thickness ( $t_m$ )



8. Calculate the chordal thickness theoretically:  $t_c = mz \sin \left( \frac{90}{z} \right)$

9. Determine the error in chordal thickness  $\Delta = t_m - t_c$



### \* Constant Chord;

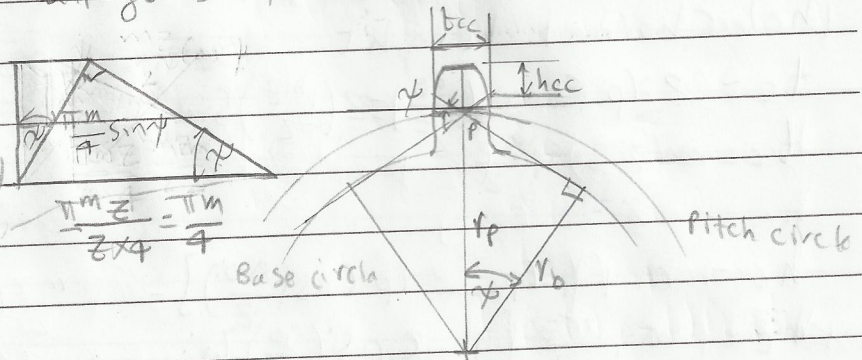
(hcc) is constant for all gears of same module

$$h_{cc} = m \cdot \frac{\pi m}{4} \cos \psi \sin \psi$$

$$h_{cc} = m \left( 1 - \frac{\pi}{4} (\cos \psi \sin \psi) \right)$$

$$h_{cc} = m \left( \frac{\pi m}{4} \sin \psi \right)$$

$$t_{cc} = \frac{\pi m}{2} \cos^2 \psi$$



### Experimental procedure:

1. Do
2. Z
3. m
4. mst
5. h<sub>cc</sub>, calculated
6. t<sub>cc</sub>, measured by gear tooth vernier
7. t<sub>cc</sub>, calculated
8. Δ = t<sub>ccm</sub> - t<sub>cc</sub>

### \* Base pitch: Normal distance between 2 invs

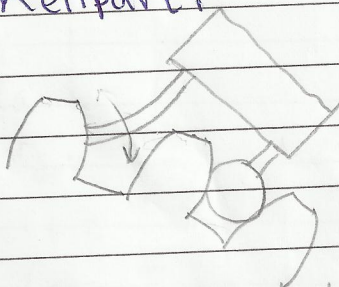
$$AB = ab - a'b'$$

↳ P<sub>b</sub> (Base pitch)

$$\overline{ac} = \overline{AC}, \overline{bc} = \overline{BC}$$

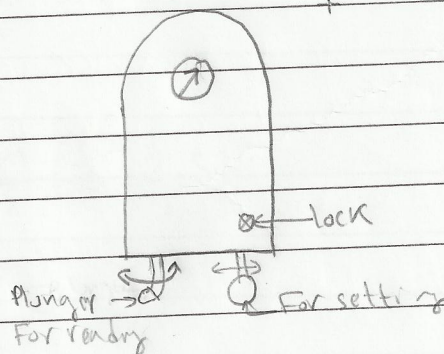
$$P_b = \frac{\pi D_b}{Z} = \frac{\pi m Z \cos \psi}{Z} = \pi m \cos \psi$$

Keilpart:



Move down or up (one direction)  
and read min. reading

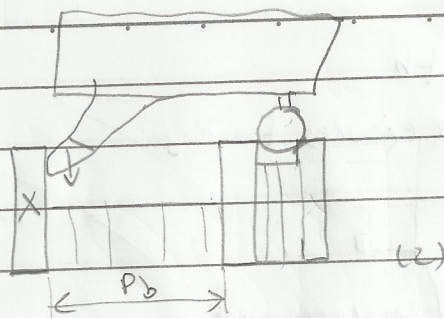
(R<sub>g</sub>)





$$P_{bm} = \overset{D_b}{D_b} + P_b$$

$$P_{bm} = (\underset{\substack{\uparrow \\ \text{on gear}}}{R_g} - \underset{\substack{\uparrow \\ \text{calculated}}}{R_{std}}) + \underset{\substack{\uparrow \\ \text{initial}}}{P_b} \quad (2)$$



### Experimental procedure:

- 1-  $D_o$
- 2-  $Z$
- 3-  $m$
- 4-  $m_{std}$
- 5- Calculate base pitch;  $P_b = \pi m \cos \psi$
- 6- Select the suitable Neilpart according to module ( $m$ )
- 7- Set the Neilpart opening to be suitable to measured gear
- 8- Take the reading on gear tooth as in (1) [least reading]
- 9- Build a block gauge combination as in (2)
- 10- Using Neilpart to take reading on combination ( $R_{std}$ ) [least reading]
- 11-  $\Delta = R_g - R_{std}$
- 12-  $P_{bm} = (R_g - R_{std}) + P_b$

— Error → Error in division during manufacturing  
                     → Error in base circle

### \* Determination of error in pitch circle:

- 1- Take reading over all teeth of gear
- 2- Consider average of readings
- 3- Compare with standard reading
  - $R_{av} = R_{std}$
  - $\Delta_{av} = R_{av} - R_{std} \rightarrow D_b = \frac{(R_{av} + P_b)Z}{\pi}$
  - $\Delta D_b = D_{bm} - m Z \cos \psi = Z \Delta_{av}$

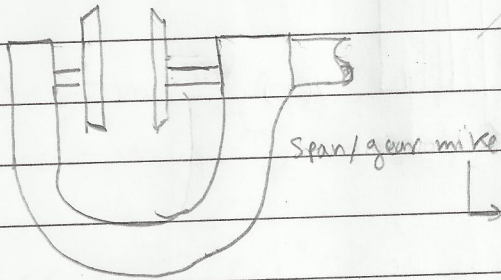


Subject. ....

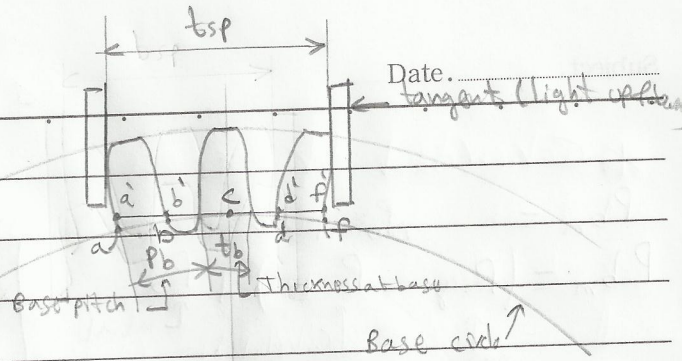
Date. ....  
Tangent (light yellow)

## \* Span length

Anvil design ensures tangency



Vernier for large gear



$$\overline{ac} = \overline{ac}$$

$$\overline{af'} = \overline{af}$$

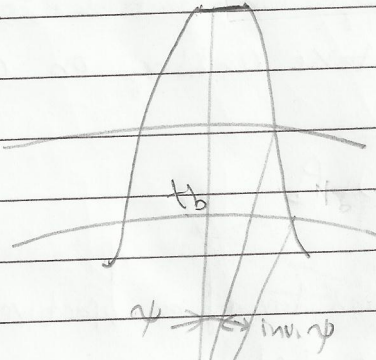
$$t_{sp} = \overline{af}$$

$$t_{sp} = (n_{sp} - 1)P_b + t_b ; n_{sp} = \frac{Z}{g} \rightarrow \text{Approximate to larger}$$

$$P_b = t_{sp_{n+1}} - t_{sp_n}$$

$$\frac{360}{20 \times 2} = g$$

$$t_{sp} = (n_{sp} - 1)\pi m \cos \psi + m \left( Z \cos \psi \operatorname{inv} \psi + \frac{\pi}{2} \cos \psi \right)$$



$$t_b = 2 \left( r_b \operatorname{inv} \psi + \frac{2\pi r_b}{4 \times Z} \right)$$

$$t_b = 2 \left( \frac{mZ \cos \psi \operatorname{inv} \psi}{2} + \frac{2\pi mZ \cos \psi}{4 \times Z \times 2} \right)$$

$$t_b = mZ \operatorname{inv} \psi \cos \psi + \frac{\pi m \cos \psi}{2}$$